A Laboratory Experiment on Transmission Line Effect

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Aims

To investigate the characteristics of a low-loss transmission line. The characteristic impedance of the transmission line under test is obtained by the experiment. The capacitance and inductance per unit length can also be found. The students are required to understand the physical meanings of voltage reflection coefficient, voltage standing wave ratio, etc.

Apparatus

1. A microwave signal generator,
2. A voltage standing wave ratio (VSWR) meter,
3. A slotted transmission line (its characteristics to be found), and
4. Purely resistive loads of various known values.

Theory

The electrical properties of a transmission line at a given frequency are completely characterized by its four distributed parameters, ***R***, ***L***, ***G***, and ***C***.

R (Ω/m) is conductor loss, ***L*** (H/m) is store magnetic energy,
G (S/m) is dielectric loss, ***C*** (F/m) is store electric energy.

A signal propagating down a transmission line is generally suffered from attenuation or reflection. The voltage and current amplitudes of the signal vary at different positions of the line and different time. They may be represented by the wave equations as follows:

$$V(z,t) = (V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z})e^{\omega t}$$

$$I(z,t) = (I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z})e^{\omega t}$$

where $\gamma = \sqrt{(\boldsymbol{R} + j\omega\boldsymbol{L})(\boldsymbol{G} + j\omega\boldsymbol{C})}$ is the *propagation constant* which governs the attenuation of different frequencies and hence signal distortion. γ is a complex number so that it is usually expressed as $\gamma = \alpha + j\boldsymbol{k}$. It represents a travelling wave which decays exponentially at a rate determined by the *attenuation constant*, α, in the positive direction of z and suffers a phase shift of ***k*** radian per unit length. ***k*** is called *phase constant*.

Characteristic impedance, $\boldsymbol{Z_0}$, is the impedance of an infinitely long transmission line. On a transmission line of infinite length, no reflected wave is present, i.e. $V_0^- = 0$ and $I_0^- = 0$. The characteristic impedance is defined as $Z_0 = V_0^+ / I_0^+$. It can be shown that

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Phase velocity, ***u***, is defined as the velocity of propagation of an equalphase front. It is largely determined by the phase constant, ***k***. Since $u = f\lambda = \lambda\omega/2\pi$, and $k = 2\pi/\lambda$, we have

$$u = \frac{\omega}{k}$$

In most practical microwave transmission lines, the loss is small. If the line is low-loss, we can assume that $R << \omega L$ and $G << \omega C$. This means that both the conductor loss and dielectric loss are small. These approximations reduce the above equations as follows:

$$\gamma = \alpha + jk \approx \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

$$Z_0 \approx \sqrt{\frac{L}{C}}$$

$$u = \frac{\omega}{k} \approx \frac{1}{\sqrt{LC}}$$

A transmission line of infinite length is just imaginary. Actually, a transmission line is terminated by a load of impedance, $\boldsymbol{Z_L}$. In this case, there are two kinds of waves that can propagate along the transmission line. They are incident (forward) travelling wave, $V_0^+ e^{-\gamma z}$, and reflected (backward) travelling wave, $V_0^- e^{\gamma z}$. At the position of the load, $z = \mathbf{0}$, the amplitude of the reflected voltage wave normalized to the amplitude of the incident voltage wave is known as the *voltage reflection coefficient*, $\boldsymbol{\Gamma_L}$. It can be calculated if $\boldsymbol{Z_L}$ is known.

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Neglecting the time-dependence and assuming low-loss transmission line, $\alpha \to 0$, the voltage wave on the line can be written as:

$$V(z) = V_0^+ e^{-jkz}\left(1 + \Gamma_L e^{j2kz}\right)$$

This equation represents a *standing wave* on the line. We can observe that the voltage on the line repeats every λ/2 with the maximum value $V_{\max} = V_0^+(1 + |\Gamma_L|)$ and the minimum value $V_{\min} = V_0^+(1 - |\Gamma_L|)$.

Voltage Standing Wave Ratio, $\boldsymbol{S}$, is defined as the ratio between $\boldsymbol{V_{\max}}$ and $\boldsymbol{V_{\min}}$:

$$S = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

As we assumed the transmission line to be **low-loss** and hence its parameters $\boldsymbol{R}$ and $\boldsymbol{G}$ considered as zero, its characteristic impedance, $\mathbf{Z_0}$, is a real number. If a **purely resistive** load is used, $\mathbf{Z_L}$ is also a real number such that

$$|\Gamma_L| = \begin{cases} \dfrac{Z_L - Z_0}{Z_L + Z_0} & ; Z_L > Z_0 \\ \dfrac{Z_0 - Z_L}{Z_L + Z_0} & ; Z_0 > Z_L \end{cases}$$

$\boldsymbol{S}$ can be re-expressed as:

$$S = \begin{cases} \dfrac{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0}} = \dfrac{Z_L}{Z_0} & ; Z_L > Z_0 \\ 1 & ; Z_L = Z_0 \\ \dfrac{1 + \frac{Z_0 - Z_L}{Z_L + Z_0}}{1 - \frac{Z_0 - Z_L}{Z_L + Z_0}} = \dfrac{Z_0}{Z_L} & ; Z_L < Z_0 \end{cases}$$

or

$$Z_L = \begin{cases} Z_0 S & ; Z_L > Z_0 \\ Z_0 & ; Z_L = Z_0 \\ Z_0 \frac{1}{S} & ; Z_L < Z_0 \end{cases}$$

We may conclude that once the transmission line is low-loss and the load is purely resistive, there is a linear relationship between $\mathbf{Z_L}$ and $\boldsymbol{S}$ for $\mathbf{Z_L > Z_0}$, and, between $\mathbf{Z_L}$ and $\mathbf{1/}\boldsymbol{S}$ for $\mathbf{Z_L < Z_0}$. If the voltage standing wave ratios, $\boldsymbol{S}$, for different values of resistive loads, $\mathbf{Z_L}$, are measured, the points can be plotted on a special graph as shown in Fig.1. The vertical axis is $\mathbf{Z_L}$. The horizontal axis is divided into two regions: the right-hand side of unity is $\boldsymbol{S}$ region which is for $\mathbf{Z_L > Z_0}$, while the left-hand side of unity is $\mathbf{1/}\boldsymbol{S}$ region which is for $\mathbf{Z_L < Z_0}$. Since $\mathbf{Z_0}$ is unknown, we do not know whether the load impedance is greater than or less than $\mathbf{Z_0}$. The method is that when $\mathbf{Z_L}$ increases, $\boldsymbol{S}$ also increases, the point should be on the S region. If $\boldsymbol{S}$ decrease as $\mathbf{Z_L}$ increases, the point should be on the $\mathbf{1/}\boldsymbol{S}$ region. By this way, a continuous straight line can be plotted. The slope of the line is the characteristic impedance of the transmission line. It is interesting that the intercept on the vertical line $\boldsymbol{S} = 1$ is also the characteristic impedance.

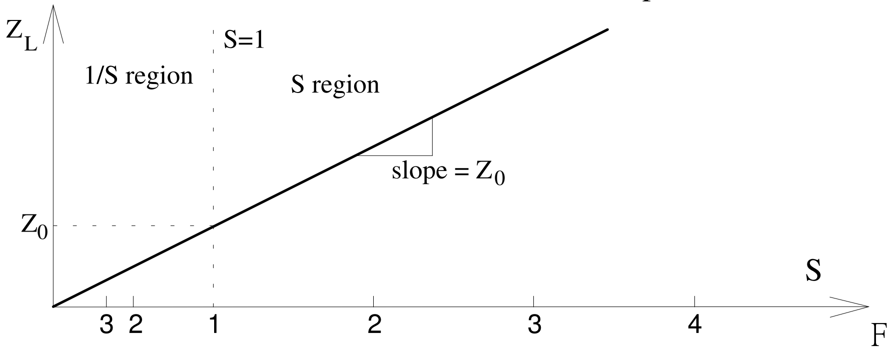

Fig. 1

On the other hand, if the wave length, λ, and the frequency, f, are known, we can obtain the parameters, $\boldsymbol{L}$ and $\boldsymbol{C}$, of the transmission line from the following set of equations:

$$\begin{cases} u = f\lambda = 1/\sqrt{LC} \\ Z_0 = \sqrt{L/C} \end{cases}$$

then, we have

inductance per unit length, $L = Z_0 / f\lambda$ (H/m), and capacitance per unit length, $C = 1 / f\lambda Z_0$ (F/m).

Measurements

The apparatus shown in Fig. 2, a *VSWR meter* is used to measure the voltage standing wave ratio on a *slotted line*. Slotted line is a transmission line configuration that allows the sampling of the electric field amplitude of a standing wave on a terminated line.

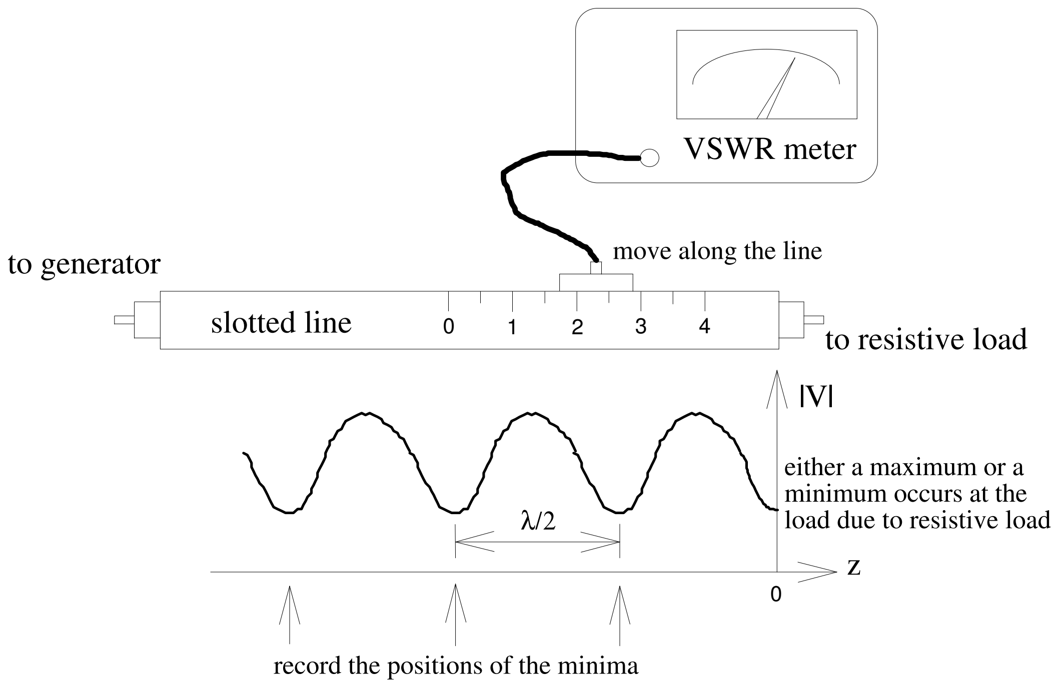

Fig. 2

When measuring the VSWR using the VSWR meter, the probe is moved along the line and the positions of the voltage minima are recorded. The distance between two adjacent minima is λ/2, and then the value of wavelength can be found.

In principle, voltage maxima locations could be used as well as voltage minima positions, but that voltage minima are more sharply defined than voltage maxima and so usually result in greater accuracy. If the value of the resistive load is close to that of the characteristic impedance of the line, the difference between the values of voltage maxima and minima is so small that it is difficult to accurately determine the minima positions. Discard that load and choose another one.

Procedure

1. Set up the apparatus as shown in Fig. 2. Select a suitable frequency of the microwave generator (frequencies between 2 to 5GHz are recommended). Record the selected frequency.
2. Connect one of the loads, $\boldsymbol{Z_L}$, to the slotted line.
3. Move the probe of the VSWR meter along the line. Record the positions of the voltage minima and the value of VSWR, $\boldsymbol{S}$.
4. Calculate the distance between the minima positions and hence the wavelength, λ.
5. Repeat from step 2 using another load until all the loads have been used.
6. Plot $\boldsymbol{Z_L}$ against $\boldsymbol{S}$ on the graph as shown in Fig. 1.
7. Determine the characteristic impedance, $\boldsymbol{Z_0}$, from the slope of the curve on the graph.
8. Take the average of the values of λ's obtained from step 4.
9. Calculate the values of $\boldsymbol{L}$ and $\boldsymbol{C}$ using the results from steps 7 and 8.